Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind,
Pune – 16
Odd Semester Examination: October 2024
Faculty: Science and Technology

Program: BSc Gen03 **Semester: V**
Program (Specific):B.Sc. **Course Type: Core**
Class: T.Y.B.Sc (Mathematics) **Max. Marks: 35**
Name of the Course: Group Theory **Course Code: 24-MT-353**
Paper no.: III **Time: 2Hrs**

Instructions to the candidate:

1) ***There are 3 sections in the question paper. Write each section on separate page.***
2) ***All sections are compulsory.***
3) ***Figures to the right indicate full marks.***
4) ***Draw a well labelled diagram wherever necessary.***

SECTION: A

Q1) Solve any five of the following. **(10 Marks)**

a) Let * be a binary operation on $\mathbb{Z}$, defined as a*b = a + b - 1. Find the identity element with respect to *.
b) Find all abelian groups up to isomorphism of order 30.
c) Define a cyclic group.
d) Find all orbits of

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix} \text{ in } S_{8}.$$

e) Is $\mathbb{Z}_4$ isomorphic to Klien's 4 group ? Justify.
f) Find the order of $(\bar{8}, \overline{10})$ in $\mathbb{Z}_{12} \, X \, \mathbb{Z}_{18}$.
g) Find the cyclic subgroup $<\rho_1>$ of symmetric group S_3 where $\rho_1 = (1, 2, 3)$.

SECTION: B

Q.2) Solve any three of the following. **(Marks 15)**

a) If G is a group and if a ∈ G, then show that H={ a^n / n∈ $\mathbb{Z}$ } forms a subgroup under multiplication.

b) Compute the factor group $\mathbb{Z}_4 \, X \, \mathbb{Z}_{6.}$ / < ($\bar{1}$, $\bar{1}$)>.

c) Consider

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 & 7 & 8 & 1 & 2 \end{pmatrix}, \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 5 & 6 & 7 & 1 & 8 & 5 \end{pmatrix}$$

i) Compute $\sigma \, \tau$

ii) Compute $\sigma\sigma^{-1}$.

iii) Write σ as product of disjoint cycles.

iv) Determine whether σ is even or odd ?

d) Find all subgroups of $< \mathbb{Z}_{12} , +_{12} >$. Hence draw its subgroup diagram.

e) Let G be the set of all real numbers except -1. Define * on G by

a*b = a+ b+ ab. Show that <G , * > is a group.

SECTION: C

Q.3) Solve any one of the following. (Marks 10)

a) Prove that M is a maximal normal subgroup of group G if and only if G/M is simple.

b) i) Let ∅ be a homomorphism of a group G into a group G′. If e is an identity element in G then prove that $\emptyset(e)$ is the identity element in G′. Also prove that if a ∈ G then $\emptyset(a^{-1}) = \emptyset(a)^{-1}$.

ii) Prove that every group of prime order is cyclic .

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